

Context Free languages

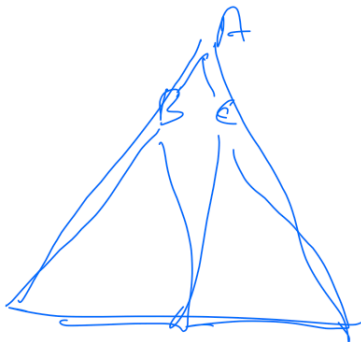
Closure Properties

Closed under

- union
- Kleene star
- intersection with regular lang
- homomorphism
- substitution (*)
- concatenation
- reverse

Not closed under

- intersection
- complementation



L

$$A \rightarrow BC$$

$$A^x \rightarrow \underbrace{C^x} \underbrace{B^x}$$

$$w = uv$$

$$w^x = v^x u^x$$

- homomorphism

$$h: a \rightarrow wa$$



$$L \rightsquigarrow G$$

$$h(L)$$

replace terminal
 a with $h(a)$
 on the RHS of production.

- inverse homomorphism

$$h: \{a, b\}^* \rightarrow \{0, 1\}^*$$

$$L \subseteq \{0, 1\}^* \text{ CFL, PDA } A_2$$

$$h^{-1}(L) \subseteq \{a, b\}^*$$

$$a \mapsto \underline{1011} \quad b \mapsto 01$$

$$q \cdot \underline{\alpha_1 \alpha_2} \xrightarrow{\quad} \underline{B_1 \alpha_1 B_2 B_3} \xrightarrow{\quad} q'$$



$h(u)$ $h(ua)$
 simulating A_2 on $h(w)$

Substitution

$$s: a \mapsto L_a \subseteq \Sigma^*$$

$$b \mapsto L_b$$

$$w = a_1 a_2 \dots a_n$$

$$s = L_{a_1} \cdot L_{a_2} \dots L_{a_n} \subseteq \Sigma^*$$

$$= \{w_1 w_2 \dots w_n \mid w_i \in s(a_i)\}$$

a b a a
 $s(a) \cdot s(b) \cdot s(a) \cdot s(a)$
 $L_a L_b L_a L_a$

$L \subseteq \{a, b\}^*$
 $s(L) = \bigcup_{w \in L} s(w)$

If L is CF, $\underline{s(a)}$ (L_a) is CF for all $a \in \Sigma$
 then $s(L)$ is CF

$G_a (S_a, \dots)$ $L_a s(a)$

$G (S, \dots)$ L

$s(L)$ $G' =$

$A \rightarrow @$

$A \rightarrow S_a$

Rational sets

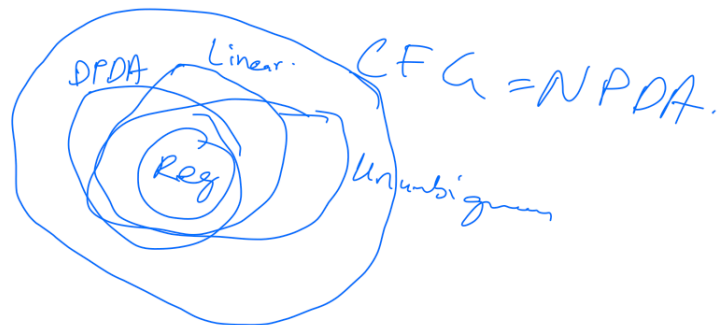
- finite languages, (CFL)
- closed under concatenation
union

$\phi \mid a \mid e_1 + e_1 \mid e_1 \cdot e_2 \mid e^*$ ^{state.}

Deterministic PDA $\xrightarrow{\neq}$ NPDA.

Reg $\xrightarrow{\neq}$ Linear CFG $\xrightarrow{\neq}$ CFG

Unambiguous grammars



Algorithmic Questions.

- Non emptiness -

Input = G a CFG

Qn $L(G) \neq \emptyset?$

$S \rightarrow AB$ a, A, B
 $A \rightarrow a$
 $B \rightarrow A$

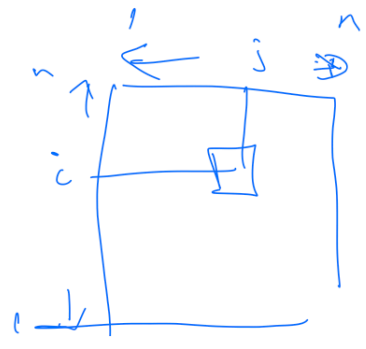
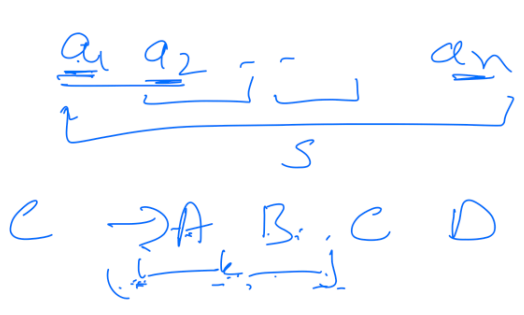
"Useful" set of NT: those that can generate some word

Terminals are useful
~~if~~ while ("Useful" has changed since last iteration
 { if $\exists A \rightarrow (\text{Useful})^*$
 then $\text{Useful} \leftarrow \text{Useful} \cup \{A\}$

Membership

I/p: $G, w \in \Sigma^*$
 Qn: $w \in L(G)?$

Cocke-Younger-Kasami (CYK)
 $O(n^3)$
 n = length of the word



$A \rightarrow a$ /

$B \rightarrow a$

Complexity of conversions: Polynomial.